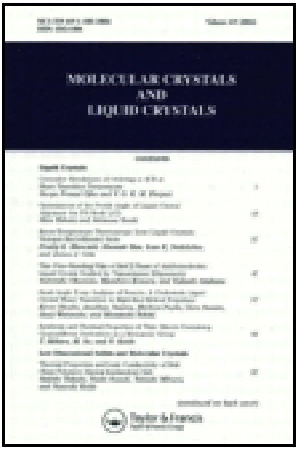

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Interchain Coupling in Quasi-One-Dimensional Superconductors: Homogeneous Coupling and Cross-Linking

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INTERCHAIN COUPLING IN QUASI-ONE-DIMENSIONAL SUPERCONDUCTORS: HOMOGENEOUS COUPLING AND CROSS-LINKING

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In quasi-one-dimensional superconductors a finite amount of electron tunneling between chains is essential to couple the superconducting order parameters on adjacent chains and to obtain thereby a nonzero transition temperature. This paper uses a microscopically derived Ginzburg-Landau theory to investigate the relation between the electron interchain tunneling and the order parameter coupling. Both a homogeneously coupled system and chains cross-linked by randomly distributed short circuits are considered. Application of the theory to $(TMTSF)_2PF_6$ shows that a reasonable concentration of short circuits may considerably increase the interchain coupling and may therefore explain the large increase of the transition temperature observed in recent experiments with GaSb-contacts.

INTRODUCTION

Quasi-one-dimensional organic superconductors of the type $(TMTSF)_2X$ [1-5] are characterized by a large anisotropy of their normal electronic properties, as shown both by transport[6] and optical[7] measurements: the electronic effective mass is of the order of the free electron mass for motion along the organic stacks (a-direction), whereas it is bigger by a factor up to 2000 in the b- and c-directions. Due to this anisotropy there is only a relatively weak coupling between the superconducting

[+]Adress from november 1981: Institut von Laue-Langevin, 38042 Grenoble, France order parameters on adjacent chains[8]. It is well known that for vanishing interchain coupling (i.e. for a strictly one-dimensional system) thermal fluctuations destroy long-range order at any finite temperature[9]. If there is a weak interchain coupling, a phase transition into a long-range-ordered superconducting state may occur at a finite temperature T_c. However, owing to the strong fluctuations, this T_c will be considerably lower than the temperature T_c^o where mean-field theory (which neglects the thermal fluctuations of the order parameter) predicts the onset of long-range order. In $(TMTSF)_2PF_6$ ($T_c \simeq 1K$ at $P=11kbar$) the very large and unusually temperature dependent conductivity has been attributed to superconducting fluctuations[8]. Considerable additional evidence for the existence of superconducting fluctuations over a large temperature region comes from measurements of the density of states by tunnel experiments[10], showing a large pseudogap (i.e. a superconducting gap washed out by fluctuations) up to at least 10K.

Very recent experimental results[11] indicate that the T_c of $(TMTSF)_2PF_6$ may be increased to 12K by evapoating GaSb onto the organic crystal. A possible explanation of this effect is an increase of the interchain coupling due to short circuits between adjacent chains which could be produced by diffusion of antimony atoms in between the chains. A larger interchain coupling due to this cross-linking suppresses fluctuations and therefore increases T_c.

It is the purpose of the present paper to estimate the effect of interchain coupling on the superconducting transition temperature and to elucidate the possibility of increasing the interchain coupling by cross-linking the chains via randomly distributed short circuits or "bridges". In the following chapter a Ginzburg-Landau model for a quasi-one-dimensional superconductor is intoduced and the depression of T_c due to fluctuations is discussed. The coefficients in the Ginzburg-Landau energy functional when only the normal, homogeneous tunneling between chains is present are calculated in the third chapter, starting from a microscopic model Hamiltonian. In the fourth chapter the effect of cross-linking on the interchain coupling is considered. In the last chapter the model is discussed and a numerical example is given, showing that a moderate concentration of bridges may indeed lead to a large increase of T_c in $(TMTSF)_2PF_6$.

GINZBURG-LANDAU MODEL

We consider a rectangular array of chains oriented along the z-axis. Each chain is described by a complex order parameter $\psi_{mn}(z)$, where m and n number the chains in the x- and y-directions, respectively. The free energy functional of the system is assumed to be of the Ginzburg-Landau form

$$F = \sum_{mn} \int dz \ \Big(a|\psi_{mn}|^2 + b|\psi_{mn}|^4 + c\left|\frac{\partial\psi_{mn}}{\partial z}\right|^2 - 2\lambda_x \operatorname{Re}(\psi^*_{mn}\psi_{m+1,n}) - 2\lambda_y \operatorname{Re}(\psi^*_{mn}\psi_{m,n+1})\Big) \ . \qquad (1)$$

As usual we set approximately $a=a'(T/T_c^o-1)$, and a',b,c,λ_x, and λ_y are positive constants to be determined in the following chapters. The first three terms in F describe the properties of a single chain, the last line is the interchain coupling.

The transition temperature in a system desribed by eq.(1) has been calculated by Scalapino et al.[12] in the limit of small interchain coupling, using exact results[13] for the single chain problem and a mean-field approximation for the interchain coupling. In this approach T_c is given by

$$1 - 2(\lambda_x+\lambda_y)\chi_{1D}(0)\Big|_{T=T_c} = 0 \ , \qquad (2)$$

where $\chi_{1D}(q)$ is the order parameter susceptibility of a chain:

$$\chi_{1D}(q) = \frac{1}{T}\int dz \ e^{iqz}\langle\psi(z)\psi(0)\rangle \ . \qquad (3)$$

At sufficiently low temperature fluctuations of the amplitude of the order parameter are frozen out and only phase fluctuations are important, so that one has

$$\chi_{1D}(q) = \frac{|a|}{Tb}\,\frac{1}{1+\xi_1^2q^2} \ , \quad \xi_1 = \frac{2c|a|}{Tb} \ , \qquad (4)$$

and inserting this into eq.(2) we obtain[14]

$$T_c = \frac{2|a|}{b}\sqrt{c(\lambda_x+\lambda_y)} \ . \qquad (5)$$

The square root dependence of T_c on the interchain coupling agrees with general arguments of Barisic and Uzelac[15].

The result (4) for the order parameter susceptibility and therefore eq.(5) are valid if only phase fluctuations are important. A quantitative estimate of the temperature region below T_c^o where amplitude fluctuations are still important (i.e. where eq.(4) is incorrect) is given by the Ginzburg[16] critical temperature region which in one dimension is[13]

$$\Delta T = 2T_c^o \left(\frac{bT_c^o}{a'^{3/2}c^{1/2}}\right)^{2/3} . \qquad (6)$$

The above treatment is valid for weak interchain coupling. On the other hand, it is of considerable interest to know how much interchain coupling is necessary to have a transition temperature near to the mean-field one. A quantitative criterion may be obtained from the first order correction to the mean-field transition temperature from thermal fluctuations, calculated in a self-consistent Hartree approximation[17,18]. To this purpose we go over to the Fourier transform of the order parameter:

$$\psi_{mn}(z) = \frac{1}{\sqrt{LN_\perp}} \sum_k \psi_k e^{i(k_\perp R_{mn}+k_z z)} , \; R_{mn} = (md_x, nd_y) , \qquad (7)$$

where L is the length of the system, $N_\perp$ is the number of chains, d_x and d_y are the interchain distances, $k_\perp$ and k_z are the perpendicular and paralell components of the wavevector k, and here and in the following the summation over k is over the first Brillouin zone. From eq.(7) one obtains

$$F = \sum_k \omega_k^2 |\psi_k|^2 + \frac{b}{LN_\perp} \sum_{kpq} \psi_k^* \psi_p^* \psi_q \psi_{k+p-q} ,$$

$$\omega_k^2 = a + ck_z^2 - 2\lambda_x \cos(k_x d_x) - 2\lambda_y \cos(k_y d_y) . \qquad (8)$$

In the Hartree approximation one sets

$$\frac{b}{LN} \sum_{kpq} \psi_k^* \psi_p^* \psi_q \psi_{k+p-q} \simeq \frac{2b}{LN} \sum_q \left\langle |\psi_q|^2 \right\rangle \sum_k |\psi_k|^2$$

$$= \Sigma(\omega_o) \sum_k |\psi_k|^2 , \qquad (9)$$

where the last equation defines the selfenergy Σ. To be self-consistent one has to replace ω_o^2 by $\bar{\omega}_o^2$ which is the solution of

$$\bar{\omega}_o^2 = \omega_o^2 + \Sigma(\bar{\omega}_o) . \qquad (10)$$

The transition temperature is given by the condition $\bar{\omega}_o(T_c)=0$, so that one has to evaluate $\Sigma(0)$. The form (8) for ω_k leads to quite complicated integrals. However, for $\bar{\omega}_o=0$ the most important contribution to Σ comes from long-wavelength fluctuations, so that one may expand ω_k^2 around k=0 and retain only terms up to second order. This results in

$$\Sigma(0) = \frac{Tb}{\pi\sqrt{c}}\, f(\lambda_x,\lambda_y) \;,$$
$$f(\lambda_x,\lambda_y) = \frac{1}{\sqrt{\lambda_y}}\,\mathrm{arsinh}\sqrt{\frac{\lambda_y}{\lambda_x}} + \frac{1}{\sqrt{\lambda_x}}\,\mathrm{arsinh}\sqrt{\frac{\lambda_x}{\lambda_y}} \quad . \tag{11}$$

Neglecting the (usually small) contribution of λ_x,λ_y to ω_o^2 we obtain from the condition $\omega_o(T_c)=0$ and eqs.(10) and (11)

$$T_c = T_c^o \left(1 + \frac{bT_c^o}{\pi\sqrt{ca'}} f(\lambda_x,\lambda_y)\right)^{-1} \;, \tag{12}$$

i.e. the denominator of this expression has to be small if T_c has to be near T_c^o. We finally remark that for weak interchain coupling (when the Hartree approximation is not expected to be very accurate) for $\lambda_x=\lambda_y$ the results (5) and (12) differ by a factor less than 2, whereas in the limit λ_x=const., $\lambda_y\to 0$ eq.(5) gives T_c=const. and eq.(12) leads to $T_c\to 0$.

MICROSCOPIC MODEL: HOMOGENEOUS INTERCHAIN COUPLING

To derive the coefficients in F microscopically we consider the model Hamiltonian

$$H = H_o + H_{int} + H_{imp} \;, \tag{13a}$$
$$H_o = \sum_{k\sigma} \varepsilon_o(k) a^+_{k\sigma} a_{k\sigma} \;,$$
$$\varepsilon_o(k) = v_F(|k_z|-k_F) - 2t_x\cos(k_x d_x) - 2t_y\cos(k_y d_y) \;, \tag{13b}$$
$$H_{int} = -\frac{g}{N}\sum_{kpq\sigma} a^+_{k\sigma} a^+_{p,-\sigma} a_{q,-\sigma} a_{k+p-q,\sigma} \;, \tag{13c}$$
$$H_{imp} = \frac{u}{N}\sum_{kpi\sigma} a^+_{k\sigma} a_{p\sigma} \exp\left(i(k-p)R_i^0\right) \quad . \tag{13d}$$

Here the $a_{k\sigma}$ are annihilation operators for electrons of wavevector k and spin σ. N is the number of sites in the system, H_o is the single-electron energy operator, where for the transverse (x-,y-) directions we assume a tight-binding form with transfer integrals t_x,t_y, respectively. The Fermi velocity is related to the longitudinal transfer integral t_z (which we assume to be much larger than t_x,t_y) by $v_F=2t_z d_z \sin(k_F d_z)$, the longitudinal effective mass of the electrons is $m=k_F/v_F$, and the d_α are the lattice constants in the three directions. The single electron energy ε_o leads to an open Fermi surface in the transverse directions. H_{int} is the BCS-type attractive electron-electron interaction which leads to superconductivity, and H_{imp} describes the scattering of electrons by impurities distributed over the sites $\{R_i^o\}$.

Starting from the model Hamiltonian, eq.(13), and following closely the method of Gorkov[19] we can calculate the coefficients of the free-energy functional. In the Born approximation the impurity averaged normal state Green's function is

$$\mathcal{G}(k,\omega_n) = \left(i(\omega_n+\mathrm{sign}(\omega_n)\frac{1}{2\tau}) - \varepsilon_o(k)\right)^{-1} , \qquad (14)$$

where $\omega_n=(2n+1)\pi T$ is the fermion Matsubara frequency, and the electronic lifetime parameter is given in terms of the impurity concentration n^o by $(2\tau)^{-1}=d_z n^o u^2/v_F$. Now, in reciprocal space the Ginzburg-Landau equation reads

$$\left(\frac{1}{g} - Q(k)\right)\psi_k + \frac{B}{LN}\sum_{pq}\psi^*_p\psi^*_q\psi_{k+p-q} = 0 . \qquad (15a)$$

Here Q(k) is the Cooper pair susceptibility:

$$Q(k) = \frac{T}{N}\sum_{np}K_n(k+p,p) . \qquad (15b)$$

K_n is the impurity average over a pair of Green's functions which, including ladder-type vertex corrections[19], is given by the equation

$$K_n(k+p,p) = \mathcal{G}(k+p,-\omega_n)\,\mathcal{G}(p,\omega_n)$$
$$\times\left(1+\sigma(k)\frac{1}{N}\sum_q K_n(k+q,q)\right) , \quad \sigma(k) = n^o u^2 . \qquad (15c)$$

This integral equation is separable and leads to

$$\frac{1}{N}\sum_p K_n(k+p,p) = \frac{1}{N}\sum_p \mathcal{G}(k+p,-\omega_n)\,\mathcal{G}(p,\omega_n)$$
$$\times\left(1 - \frac{\sigma(k)}{N}\sum_p \mathcal{G}(k+p,-\omega_n)\,\mathcal{G}(p,\omega_n)\right)^{-1} . \qquad (15d)$$

As discussed in the previous chapter near the transition temperature only long-wavelength fluctuations are important, so that we may limit ourselves here to terms up to second order in k. The mean-field transition temperature is given by the condition 1/g-Q(0)=0. From eqs.(15) then we obtain

$$\frac{1}{g} - Q(k) = \frac{d_z}{\pi v_F}\left(\ln(T/T_c^o) + \frac{7\zeta(3)}{16\pi^2T^2}\chi\left((2\pi\tau T)^{-1}\right)\right.$$
$$\left.\times(v_F^2k_z^2 + 2t_x^2d_x^2k_x^2 + 2t_yd_y^2k_y^2)\right) . \qquad (16)$$

$\zeta(x)$ is Riemann's zeta function and $\chi(x)$ is the Gorkov function

$$\chi(x) = \frac{8}{7\zeta(3)x}\left(\frac{\pi^2}{8} + \frac{1}{2x}\left(\psi(\tfrac{1}{2}) - \psi(\tfrac{1+x}{2})\right)\right) , \qquad (17)$$

and $\psi(x)$ is the digamma function[20]. The coefficient B is given by an averadged product of four Green's functions[19].

The Ginzburg-Landau equation is obtained by requiring the free-energy to be stationary. Normalizing the order parameter so that c=1/2m and comparing the small-k expansion of ω_k with eq.(16) it follows from eqs.(15a) and (16)

$$a = \frac{8\pi^2T^2}{7\zeta(3)mv_F^2\chi\left((2\pi\tau T)^{-1}\right)}\ln(T/T_c^o) = a'\ln(T/T_c^o) \;, \tag{18a}$$

$$\lambda_\alpha = t_\alpha^2/(mv_F^2) \quad , \qquad \alpha=x,y \quad , \tag{18b}$$

and the fourth order coupling is

$$b = a'/(2k_F\chi\left((2\pi\tau T)^{-1}\right)) \quad . \tag{18c}$$

As already pointed out in ref.8 the transverse couplings are proportional to the square of the tunneling integrals, or inversely proportional to the square of the transverse effective masses. Contrary to the present case of an open Fermi surface, for a closed Fermi surface these couplings are inversely proportional to the masses themselves.

We note that setting approximately $T=T_c^o$ in eqs.(18) one obtains the Ginzburg critical temperature region

$$T = 1.6T_c^o\chi\left((2\pi\tau T)^{-1}\right)^{-1/3} \quad , \tag{19}$$

i.e. even for a pure system (χ=1) the critical region is larger than T_c^o itself. Though the approximation $T\simeq T_c^o$ is obviously inconsistent over such a wide temperature region, this result nevertheless indicates that amplitude fluctuations are important even far below T_c^o, so that the approximations leading to eq.(5) are only valid for very weak coupling.

On the other hand, if the interchain coupling is sufficiently strong so that the Hartree approximation becomes valid, the lowering of T_c due to fluctuations can be expressed in terms of an effective interchain coupling J:

$$J = \sqrt{2t_xt_y}\,\chi\left((2\pi\tau T)^{-1}\right)\left(\sqrt{\frac{t_x}{t_y}}\,\mathrm{arsh}\frac{t_y}{t_x}+\sqrt{\frac{t_y}{t_x}}\,\mathrm{arsh}\frac{t_x}{t_y}\right)^{-1} \quad , \tag{20a}$$

so that from eq.(12) one obtains

$$T_c = JT_c^o/(J+T_c^o) \tag{20b}$$

MICROSCOPIC MODEL: CROSS-LINKING

Recent experiments[11] indicate that the transition temperature of $(TMTSF)_2PF_6$ under pressure may be considerably increased by evaporation of a GaSb-layer onto the crystal. A possible explanation[21] of this effect is that antimony atoms diffuse into the crystal. Due to their small size, compared to the size of a TMTSF molecule, these atoms may come very close to the organic molecular stacks. The atomic orbitals then have a considerably larger overlap with adjacent molecules than the direct TMTSF-TMTSF overlap, i.e. the antimony atoms would create short-circuits or "bridges" between the chains. In this chapter such a bridge is represented by a local transfer integral t. In order to gain some physical insight in the effect of randomly distributed bridges on the coupling between the superconducting order parameters we shall first investgate the cross-linking effect between two isolated chains.

Two Cross-Linked Chains

We consider a model Hamiltonian

$$H_{II} = H_1 + H_2 + H_{1-2} \quad . \tag{21}$$

Here H_1 and H_2 describe the electrons on the individual chains, similar to H of eq.(13), however without impurity scattering (u=0). The "cross-linking Hamiltonian" H_{1-2} describes electron tunneling from one chain to the other at randomly distributed sites $\{R_i\}$ and has the form

$$H_{1-2} = \frac{t}{N}\sum_{kp\sigma i} e^{i(k-p)R_i}(a^+_{1k\sigma}a_{2p\sigma} + a^+_{2k\sigma}a_{1p\sigma}) \quad . \tag{22}$$

The operator $a_{\alpha k\sigma}$ (α=1,2) refers to electrons on chain α. Due to the lack of translational invariance the single-electron Green's function is not diagonal in the wavenumber, and to lowest order in t one has

$$\mathcal{G}(\alpha k,\beta p,\omega_n) = \mathcal{G}^o(k,\omega_n)\Big[\delta_{kp}\delta_{\alpha\beta} + (1-\delta_{\alpha\beta})\mathcal{G}^o(p,\omega_n) \times\frac{t}{N}\sum_i e^{i(k-p)R_i} + O(t^2)\Big] \quad . \tag{23}$$

$\mathcal{G}^o$ is the single-electron Green's function for t=0, and α,β are chain indices. To this order the intrachain part remains diagonal in k, and only the interchain part has nondiagonal components. Correspondingly, in the Ginzburg-Landau equation lerived from H_{II} (retaining only the lowest order in the superconducting order parameters):

$$\left(\frac{1}{g} - Q(k)\right)\psi_1(k) - \sum_q Q_1(k,q)\psi_2(q) = 0 \tag{24}$$

only the interchain coupling function Q_1 has nonvanishing components for $k=q$. Q_1 is given by

$$\begin{aligned} Q_1(k,q) &= \frac{T}{N}\sum_{prn} \mathcal{G}(2,k+p,1,q+r,-\omega_n)\,\mathcal{G}(1r,2p,\omega_n) \\ &= \frac{Tt^2}{N^3}\sum_{prnij} \mathcal{G}^o(k+p,-\omega_n)e^{i(k+p-q-r)R_i}\mathcal{G}^o(q+r,-\omega_n) \\ &\quad \times \mathcal{G}^o(r,\omega_n)e^{i(r-p)R_j}\mathcal{G}^o(p,\omega_n) \quad . \end{aligned} \tag{25}$$

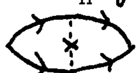
Here we are interested in the averadged (over the positions) value of Q_1. Averaging each bridge independently one has

$$\overline{\exp(iqR_i)} = \delta_{q,0} \quad . \tag{26}$$

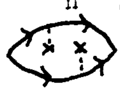
Terms with $i=j$ and with $i \neq j$ lead to contributions Q_{11} and Q_{12} to the averaged Q_1, respectively:

$$\overline{Q_1(k,q)} = \delta_{kq}Q_1(k) = \delta_{kq}\left(Q_{11}(k)+Q_{12}(k)\right) \quad , \tag{27a}$$

$$Q_{11}(k) = \frac{Tt^2n}{N^2}\sum_n\left[\sum_p \mathcal{G}^o(k+p,-\omega_n)\mathcal{G}^o(p,\omega_n)\right]^2$$

$=$  , (27b)

$$Q_{12}(k) = \frac{Tt^2n^2}{N}\sum_{pn}\mathcal{G}^o(k+p,-\omega_n)^2\mathcal{G}^o(p,\omega_n)^2$$

$=$  , (27c)

where n is the concentration of bridges. For long-wavelength configurations only $Q_1(0)$ is important. From (27) one obtains

$$Q_{11}(0) = \frac{nt^2d_z^2}{4Tv_F^2} , \quad Q_{12}(0) = \frac{7\zeta(3)n^2t^2d_z}{8\pi^3T^2v_F} \tag{28}$$

leading to a total coupling factor between the chains

$$Q_1(0) = \frac{7\zeta(3)nt^2d_z}{8\pi^3T^2v_F}\left(\frac{d_z}{\xi_o} + n\right) \quad , \quad \xi_o = \frac{7\zeta(3)v_F}{2\pi^3T} \tag{29}$$

Physically, the coupling of the order parameters is due to tunneling of Cooper pairs through the bridges. The Q_{12}-contribution comes from processes in which the two electrons tunnel at different sites. The propability of a single electron to tunnel is proportional to nt, so that one has $Q_{12} \propto (nt)^2$. On the other hand, in the Q_{11}-process both electrons tuneel at the same site. The propability for this process is directly proportional to the number of sites and, as two electrons are involved, to t^2, leading to $Q_{11} \propto nt^2$. However, before the tunneling process the Cooper pair has to move to the site of the bridge, whereas in the Q_{12}-process the two electrons move individually to the two bridges (i.e. the Q_{12}-process involves the relative motion of the two electrons). It is to this difference that we attribute the factor d_z/ξ_0 in Q_{11}.

Instead of using the bridge-averadged Q_1 in the Ginzburg-Landau equation another possible approach would be to calculate the coefficients of the free energy functional for a fixed bridge distribution and to average only the results derived from this free energy. Concerning the Q_{11}-coupling this leads to results exactly equivalent to the present approach. However, the proper treatment of the Q_{12}-process, which involves correlated pairs of bridges, would lead to considerable difficulties. Therefore, also in the following section we shall average directly in the microscopic equations.

Cross-Linking in a Quasi-One-Dimensional System

In an array of coupled chains the cross-linking operator is

$$H_{cr} = \frac{1}{N} \sum_{kp\sigma ij} a^+_{k\sigma} a_{p\sigma} \Big\{ t^x e^{i(k-p)R^x_i}(e^{-ip_x d_x} + e^{ik_x d_x}) + t^y e^{i(k-p)R^y_j}(e^{-ip_y d_y} + e^{ik_y d_y}) \Big\} \quad , \qquad (30)$$

which is added to H of eq.(13). Here $\{R^x_i\}$ and $\{R^y_j\}$ are the randomly distributed sites of bridges connecting adjacent chains in the x- and y-directions, respectively, and t^x, t^y are the corresponding transfer integrals.

In Born approximation the average Green's function is

$$\mathcal{G}(k,\omega_n) = \big(i(\omega_n + \mathrm{sign}(\omega_n) - \varepsilon(k)\big)^{-1} \ ,$$

$$\varepsilon(k) = v_F(|k_z| - k_F) - 2(t_x + n^x t^x)\cos(k_x d_x) - 2(t_y + n^y t^y)\cos(k_y d_y)$$

$$(2\tau)^{-1} = \frac{d_z}{v_F}(n^o u^2 + 2n^x t^{x2} + 2n^y t^{y2}) \quad , \qquad (31)$$

where n^x, n^y are the concentration of the bridges in the corresponding directions. The bridges give both an addtional contribution to the transverse bandwidth (due to the increased tunneling propability) and to the scattering time (due to the additional disorder).

As before, the Cooper pair susceptibility is given by eqs.(15b)-(15d), however, the vertex correction factor $\sigma(k)$

now reads

$$\sigma(k) = n^o u^2 + 2n^x t^{x^2} \cos(k_x d_x) + 2n^y t^{y^2} \cos(k_y d_y) \ . \quad (32)$$

Proceeding as in the third chapter, the coefficients a,b, and c are unchanged from eq.(18), and the interchain coupling coefficients are

$$\lambda_\alpha = \frac{(t_\alpha + n^\alpha t^\alpha)^2}{mv_F^2} + \frac{2\pi^3 T d_z}{7\zeta(3) m v_F^3 \chi\left((2\pi\tau T)^{-1}\right)} n^\alpha t^{\alpha^2}, (\alpha=x,y) \quad (33)$$

The first term comes from the increased transverse bandwidth of the individual electrons. This corresponds to the Q_{12}-process of the previous section. The second terms comes from the dependence of the vertex correction σ on the transveres momenta. This is the Q_{11}-process. Indeed, the diagram representing Q_{11} (eq.(27b)) is exactly the first term of the ladder graph series represented by σ. The second term in (33) has an additional factor χ^{-1} with respect to the first. The origin of this difference is similar to the point discussed at the end of the previous section: A process where both electrons tunnel at the same point (the second term in (33)) is less sensitive to loss of coherence induced by impurity scattering than a process where the two electrons tunnel at two different (and distant) points.

DISCUSSION AND NUMERICAL APPLICATION

In deriving our results various approximations have been made. Concerning the microscopic model for cross-linking, it may appear somewhat simplified to describe the coupling between chains due to an intercalated atom by a simple addtional transfer integral. In a more realistic model one would consider an additional localized atomic level of energy ε and describe the overlap of this orbital with the two adjacent chains by tunneling integrals t. However, preliminary calculations indicate that the coupling between order parameters in such a model may indeed be described by a single transfer integral t as long as $|\varepsilon-\varepsilon_F|<t$ (ε_F=Fermi energy).

The derivation of the coefficients of the Ginzburg-Landau functional involves several assumptions. First, in treating the average over the bridge sites we have neglected both the possibility of percolation type phenomena, which may become important at very low concentrations, and correlations between the positions of bridges which, however, we expect to be small at concentrations below 10-20%. Also, calculating averaged coefficients we have neglected the local change of the order parameter due to the bridges. In analogy to the case of normal impurities, however, this effect is expected to be unimportant. Finally, the derivation given here implicitly assumes a small order parameter, whereas at least at low temperature, where the order parameter has a well defined amplitude, it would be more appropriate to include this fixed amplitude in the starting Hamiltonian.

A more fundamental objection concerns the application of Ginzburg-Landau theory, which treats the order parameter as a classical variable and has long-range order at T=0, to a one-dimensional Fermi gas, which is well known to have no long-range order and power-law decaying correlation functions at T=0[22]. One should, however, notice that considering the order parameter as a quantum variable one recovers, at least qualitatively, the T=0-properties of the microscopic model[23]. It appears therefore that even in the present case Ginzburg-Landau theory provides a convenient starting point, though the necessary extensions to make contact with microscopic one-dimensional models requires further work.

In conclusion, we believe that in spite of the above discussed approximations the present model provides a basis at least for a semiquantitative understanding of the effect of cross-linking in quasi-one-dimensional superconductors. Quantitative values for the parameters of the theory are derived as follows: Plasma edge studies[7] give t_z=250meV and t_x=3meV for the longitudinal and largest transverse transfer integrals, respectively. The anisotropy of the perpendicular upper critical fields is[24] $H^x_{c2}/H^y_{c2}=15$. From the Ginzburg-Landau theory of H_{c2} of Turkevich and Klemm[25] and eq.(18b):

$$H^x_{c2}/H^y_{c2} = (d_x t_x)/(d_y t_y) \quad , \qquad (34)$$

and using the lattice constants[26] of $(TMTSF)_2PF_6$ one arrives at t_y=0.1meV. From the t_z-t_x anisotropy, ref.25, and eq.(18b) one would expect $H^z_{c2}/H^x_{c2}=40$, apparently larger than the observed value[27]. Apart from an overestimation of the anisotropy from the plasma edge data two reasons may explain the discrepancy:(i) due to the large anisotropy H^z_{c2} is highly sensitive to very small deviations from the exact z-direction[25] and may therefore be hard to determine (indeed, some experiments[28] show very large values of dH^z_{c2}/dT up to 70kG/K), (ii) preliminary calculations show that fluctuations tend to decrease the H_{c2}-anisotropy below its mean-field[25] value. In view of these difficulties we use the plasme edge results. Assuming a mean free path ($=v_F\tau$) of 250Å one obtains (with T=1K) J=1.2K. This means that even for a relatively high T^o_c one expects a T_c of the order of J, as observed.

Finally, let us consider a numerical example for cross-linking: assume a concentration of 2% of "perfect" bridges (i.e. $t^{\alpha}=t_z=250$meV) in both transverse directions. From eqs. (17),(31), and (33) one obtains $\lambda_{\alpha}=0.18$meV, where 80% of the coupling comes from the second term in (33), i.e. from the Q_{11}-process. This gives (at T=10K) J≈150K, i.e. the coupling is greatly enhanced with respect to the case without cross-linking. Assuming for $(TMTSF)_2PF_6$ $T_c^o \simeq 15$K we conclude from eq.(20b) that a moderate concentration of 4% bridges is largely sufficient to explain the observed stabilization of superconductivity[11] at T=12K.

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